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THERMODYNAMICS OF BLACK HOLES UNDER NONLINEAR ELECTRODYNAMICS (NED) CONDITIONS IN A CHARGED, NON-ROTATING ANTI-DE SITTER (ADS) SPACE

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Abstract: This study examines black hole thermodynamics in charged, non-rotating Anti-de Sitter (AdS) space influenced by Non-Linear Electrodynamics (NED). It analyzes the metric solution, Hawking temperature, and specific heat, highlighting phase transitions and critical points that affect thermodynamic stability, revealing NED's significant impact on black hole behavior.

Key words: *Black holes, thermodynamic, event horizon, NED, charge, AdS.*

Introduction. Black hole thermodynamics is a key area of study that links gravitation, quantum theory, and thermodynamics. In Anti-de Sitter (AdS) space, incorporating Non-Linear Electrodynamics (NED) adds layers of complexity, altering both the black hole's metric and thermodynamic characteristics [1-2]. This research focuses on charged, non-rotating black holes in AdS space, analyzing how NED influences their thermodynamic behavior.

The black hole's metric function plays a central role in understanding its thermodynamics [3]. This modified metric is vital for calculating thermodynamic properties like Hawking temperature and specific heat. The Hawking temperature, linked to the black hole's surface gravity, is essential in determining the thermal radiation emitted by the black hole.

The specific heat is also analyzed to assess the black hole's stability. It reveals critical points where phase transitions occur, signifying changes in the black hole's stability [4].

In this work, we adopt the signature $(-,+,+,+)$ for the space-time and utilize the unit system where $G = c = 1$. However, in our expressions related to astrophysical applications, we explicitly include the speed of light. The Latin indices span from 1 to 3 , while the Greek indices range from 0 to 3[5] .

Action Integral.

For a charged, non-rotating (static) Anti-de Sitter (AdS) black hole, the action integral is given by:

$$S = \int d^4x\sqrt{-g}\left(\frac{R-2\Lambda}{16\pi} + \mathcal{L}_{\text{NED}}\right) \tag{1}$$

where g is the determinant of the metric, R is the Ricci scalar, Λ is the cosmological constant, and $\mathcal{L}_{\text{NED}}$ represents the Lagrangian of Non-Linear Electrodynamics (NED).

For a spherically symmetric black hole, the Ricci scalar is expressed as:

$$R = -\frac{2}{r^2}\left(1 - f(r) - rf'(r)\right) \tag{2}$$

where $f(r)$ is the metric function to be determined by solving the Einstein equations.

The metric for a charged, non-rotating AdS black hole under the conditions of Non-Linear Electrodynamics is given by[6-7]:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

where the metric function $f(r)$ is

$$f(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2} - \frac{\Lambda r^2}{3} \tag{4}$$

with M being the mass of the black hole, q its charge, and Λ the cosmological constant, which is related to the AdS pressure P by

$$P = -\frac{\Lambda}{8\pi} \tag{5}$$

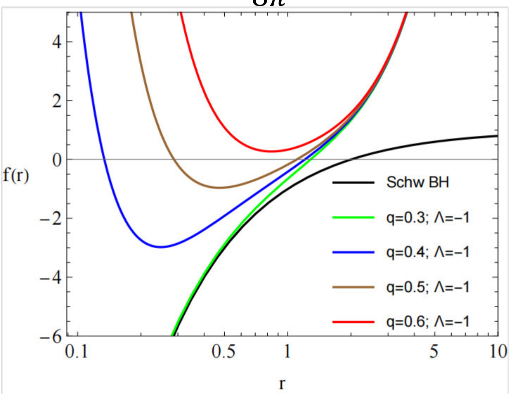

Fig. 1: Radial dependence of lapse function $f(r)$ for varying values of q and Λ.

In Fig.1 shows the behavior of the metric function $f(r)$ as a function of radial distance r, comparing different black hole charge values q with a fixed cosmological constant $\Lambda = -1$. The Schwarzschild black hole ($q = 0$) is included for reference.

Thermodynamics of AdS BH.

The Hawking temperature T of the black hole is related to the surface gravity at the event horizon r_h and can be expressed as[8]:

$$T = \frac{f'(r_h)}{4\pi} = -\frac{3q^2 - 3Mr + r^4\Lambda}{6\pi r^3} \tag{6}$$

The entropy of the black hole is proportional to the area of the event horizon, as per the Bekenstein-Hawking entropy formula:

$$S = \frac{A}{4} = \pi r_h^2 \tag{7}$$

The specific heat at constant charge C is defined as:

$$C = T\left(\frac{\partial S}{\partial T}\right) \tag{8}$$

Since $S = \pi r_h^2$, we have:

$$\frac{\partial S}{\partial r_h} = 2\pi r_h \tag{9}$$

Next, we need to differentiate the temperature T from r_h. From the temperature expression [15-16]:

$$C = \frac{2\pi r^2(3q^2 - 3Mr + r^4\Lambda)}{6Mr - 9q^2 + r^4\Lambda} \tag{10}$$

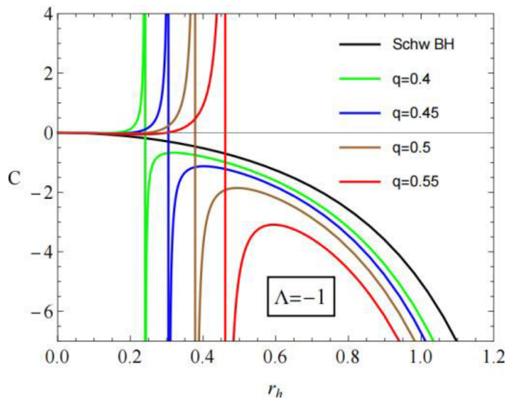

Fig. 2: The vertical axis represents the heat capacity C, while the horizontal axis represents the event horizon radius r_h

In Fig. 2, the Schwarzschild black hole shows a relatively smooth behavior with negative heat capacity, decreasing gradually as r_h increases. The appearance of these divergences for charged black holes contrasts with the smoother behavior of the Schwarzschild black hole, which does not exhibit such transitions in this region. Physically, this means that the black hole undergoes a second-order phase transition. Then, this phase transition leads the black hole to move from the stable phase ($C > 0$) to the unstable phase ($C < 0$).

***P − V* Criticality.**

The $P - V$ criticality of black holes shows an analogy to Van der Waals fluids, where phase transitions occur between small and large black holes[9]. The equation for temperature is:

$$T = \frac{f^{'}(r_h)}{4\pi} = -\frac{3q^2 - 3Mr + r^4\Lambda}{6\pi r^3} \tag{11}$$

$$P = \frac{3(q^2 - Mr + 2\pi r^3 T)}{r^4} \tag{12}$$

This formula represents the pressure P inside the black hole system, also depending on the charge q, the black hole mass M, and the temperature T[10].

$$\left(\frac{\partial P}{\partial V}\right)_T = 0, \ \left(\frac{\partial^2 P}{\partial V^2}\right)_T = 0 \tag{13}$$

To find the critical points, these two conditions are used. The first condition Eq. 13 ensures that the pressure is stationary concerning the volume V at constant temperature T. The second condition Eq. 13 ensures that the system is at a point of inflection, which corresponds to the critical point of a phase transition.

Solving these equations gives the critical radius r_c, pressure P_c, temperature T_c, and volume V_c. For a Reissner-Nordström AdS black hole, the critical values are:

$$r_c = \sqrt{6}q, \ P_c = \frac{1}{96\pi q^2}, \ T_c = \frac{\sqrt{6}}{18\pi q}, \ V_c = \frac{8\sqrt{6}\pi q^3}{3} \tag{14}$$

The equations provide the thermodynamic behavior of a charged black hole in AdS space, defining how temperature, pressure, and volume interact at critical points. These critical points are associated with phase transitions, such as the ones between small and large black holes, analogous to the liquid-gas phase transitions in conventional thermodynamics. The formulas describe the relationships between the black hole's physical parameters like mass, charge, and the cosmological constant Λ, and how these contribute to its thermodynamic properties.

Conclusion. In this paper, we investigated the thermodynamic properties and P-V criticality of charged, non-rotating black holes in AdS space under Non-Linear Electrodynamics (NED). Our findings reveal that these black holes exhibit phase transitions similar to Van der Waals fluids. Analyzing the Hawking temperature and specific heat allowed us to identify critical points, showing that NED significantly affects the black hole's thermodynamic stability and phase structure in AdS space.

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APPLICATION OF WOLFRAM MATHEMATICA IN THEORETICAL ASTROPHYSICS

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Abstract: Wolfram Mathematica has proven to be a powerful tool in astrophysics due to its extensive capabilities in symbolic computation, numerical analysis, and data visualization. Its ability to solve complex differential equations, simulate physical systems, and visualize multidimensional datasets makes it a valuable asset for theoretical and observational astrophysicists. The present paper briefly explores the various applications of Mathematica in astrophysics, including solving Einstein's field equations, modeling stellar dynamics, simulating cosmological phenomena, and analyzing data from astronomical observations. Furthermore, we highlight Mathematica's role in simplifying the intricate mathematics involved in these tasks and its contributions to advancing our understanding of the universe.

Keywords: *Wolfram Mathematica, Astrophysics, Einstein equations*

Astrophysics is a field that heavily relies on mathematical models to explain and predict cosmic phenomena. From the movement of celestial bodies to the properties of black holes and the expansion of the universe, astrophysical research requires solving highly complex equations, analyzing large datasets, and visualizing multidimensional information. Wolfram Mathematica, with its powerful symbolic and numerical computation capabilities, has become an indispensable tool in these areas