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OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

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"UZLUKSIZ TA'LIM JARAYONIDA FIZIKA, MATEMATIKA VA INFORMATIKA FANLARINING DOLZARB MUAMMOLARI VA YECHIMLARI"

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XALQARO ILMIY-AMALIY ANJUMANI

MAQOLALARI TO'PLAMI

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UDK 517.5+517.95+517.97+517.98+517.958+517.968+517.6

“Uzluksiz ta’lim jarayonida fizika, matematika va informatika fanlarining dolzarb muammolari va yechimlari” : Xalqaro ilmiy-amaliy anjuman ma’ruzalari tezisi (24-25 oktyabr 2025-yil, Samarqand, O‘zbekiston). – Samarqand, 2025. 694-b.

Ushbu to‘plam “Uzluksiz ta’lim jarayonida fizika, matematika va informatika fanlarining dolzarb muammolari va yechimlari” xalqaro ilmiy-amaliy anjuman qatnashchilarining ilmiy ma’ruzalarini quyidagi yo‘nalishlarda saqlaydi: matematika hamda uni o‘qitishning dolzarb muammolari va yechimlari; informatika hamda uni o‘qitishning dolzarb muammolari va yechimlari; fizika hamda uni o‘qitishning dolzarb muammolari va yechimlari.

Ushbu xalqaro ilmiy-amaliy anjuman O‘zbekiston Respublikasi Prezidentining 2023-yil 4-iyuldagi “Ma’muriy islohotlar doirasida oliy ta’lim, fan va innovatsiyalar sohasida davlat boshqaruvini samarali tashkil qilish chora-tadbirlari to‘g‘risida”gi PQ-200 Qarori hamda O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligining 2024-yil 27-dekabrdagi “2025-yilda xalqaro va respublika miqyosida o‘tkaziladigan ilmiy va ilmiy-texnik tadbirlar rejalarini tasdiqlash to‘g‘risida”gi 490-sonli buyrug‘i ijrosini ta’minlash maqsadida tashkil etilgan.

TAHRIRIYAT KENGASHI

NASHR UCHUN MA’SULLAR

To‘plamda joy olgan tezislarda keltirilgan ma’lumotlarning to‘g‘riligiga mualliflar javobgardirlar

- Masofaviy ta’lim: Pandemiya davrida simulyatsiyalar masofaviy ta’limda muhim vosita bo‘ldi.

PhET simulyatsiyalari keng imkoniyatlarga ega bo‘lsa-da, quyidagi muammolar mavjud:

- Texnik cheklovlar: Ba’zi maktablarda internet yoki zamonaviy kompyuterlarning yetishmasligi simulyatsiyalardan foydalanishni qiyinlashtiradi.
- O‘qituvchi malakasi: Simulyatsiyalarni darsga samarali integratsiya qilish uchun o‘qituvchilar maxsus tayyorgarlikdan o‘tishi kerak.
- Talaba faolligi: Noto‘g‘ri qo‘llanilganda, simulyatsiyalar o‘yin sifatida qabul qilinishi mumkin, bu esa o‘quv maqsadlariga erishishni qiyinlashtiradi.

Xulosa: PhET simulyatsiyalari fizika darslarida raqamli laboratoriya sifatida muhim o‘rin tutadi. Ular talabalarga fizik jarayonlarni interaktiv va vizual tarzda o‘rganish imkonini beradi, o‘quv jarayonini qiziqarli va samarali qiladi. Adabiyotlar tahlili shuni ko‘rsatadiki, simulyatsiyalar talabalarning ilmiy tushunchalarini chuqurlashtirishda va an’anaviy laboratoriyalarga muqobil sifatida muhim hissa qo‘shadi. O‘zbek ta’lim tizimida PhET simulyatsiyalarini kengroq joriy etish uchun o‘qituvchilar malakasini oshirish va texnik infratuzilmani rivojlantirish bo‘yicha qo‘shimcha choralar ko‘rish zarur.

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DYNAMICS OF TEST PARTICLES AROUND QUASI- AND NON-SCHWARZSCHILD BLACK HOLES

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Abstract: In this work, we study particle dynamics in the spacetime of quasi- and non-Schwarzschild black holes. We compare the gravitational effects of spacetime deformation parameters of both black hole solutions on innermost stable circular orbits (ISCOs) position, energy, and angular momentum of test particles at ISCO together with the energy efficiency of accretion disc in the thin Novikov-Thorn model. Also, we study frequencies of particle oscillations in radial and angular directions along circular stable orbits around both deformed black holes.

Keywords: *non-Schwarzschild black holes, quasi-Schwarzschild black holes, quasiperiodic oscillations*

1. Introduction

From the mathematical point of view, black holes (BHs) are exact solutions to the gravitational field equations. For the first time, Karl Schwarzschild found a spherically symmetric BH solution that has only one parameter total mass, as an exact vacuum solution to Einstein’s field equations in 1916.

2. “Deformed” black hole solutions

2.1. Quasi-Schwarzschild black holes

The quasi-Schwarzschild spacetime metric takes the following form

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + k(r,\theta)r^2 d\Omega^2 \tag{1}$$

where

$$g_{tt} = -\left(1 - \frac{2M}{r}\right)[1 - \epsilon F_1(1 - 3\cos^2\theta)]$$
$$g_{rr} = \frac{r}{r - 2M}[1 + \epsilon F_1(1 - 3\cos^2\theta)]$$
$$k(r,\theta) = 1 - \epsilon F_2(1 - 3\cos^2\theta)$$

2.2. Non-Schwarzschild black holes

The spacetime around a black hole obtained by Johansen and Psaltis that is a so-called non-Schwarzschild black hole in spherical coordinates has the form ([1]):

$$ds^2 = -f(r)\big(1 + h(r)\big)dt^2 + \frac{1+h(r)}{f(r)}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{2}$$

with radial functions

$$f(r) = 1 - \frac{2M}{r},\ h(r) = \sum_{k=1}^{\infty} \varepsilon_k \left(\frac{M}{r}\right)^k \tag{3}$$

where $h(r)$ stands to indicate the deviation of the spacetime from the Schwazrzschild one and ε is the deformation parameter, and it takes both positive or negative values, namely, $\varepsilon > 0$ or $\varepsilon < 0$ describing oblate and prolate deformations, respectively. If there is no deformation in the space-time $\varepsilon = 0$, the metric (3) turns Schwarzschild. The zero term is extremely small $\varepsilon0 \ll 1$ or $\varepsilon0 = 0$, and similarly $\varepsilon1 \ll M/r$ and authors of Ref.[2] have shown that $\varepsilon2 \simeq 4.6 \times 10{-}6$ using observational data of the Laser Lunar Ranging experiment, so one may neglect it. The lowest-order at $k = 3$ non-vanishing parameter $\varepsilon3 = \varepsilon$ is mentioned above as the deformation parameter [1].

3. Motion of test particles

The present section is devoted to exploring the circular orbit of test particles with mass m around both non-Schwarzschild and quasi-Schwarzshild black holes employing the Hamilton-Jacobi equation

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu} = -m^2 \tag{4}$$

One can derive the equations of motion for the test files using the following standard separable form of the action S as,

$$S = -Et + L\varphi + S_r + S_\theta \tag{5}$$

where E and L are energy and the angular momentum of the test particle i.e. integrals of the motion. One may easily find the effective potential of the test particles for circular motion $(\dot{r} = 0)$in a fixed plane $(\dot{\theta} = 0)$in the following form

$$V_{eff} = -g_{tt}\left(1 + \frac{\mathcal{L}^2}{g_{\phi\phi}}\right) \tag{6}$$

The values of specific angular momentum of test particles corresponding to circular orbits can be obtained by solving the equation $\partial_r V_{eff}(r) = 0$given as,

$$\mathcal{L}_c^2 = \frac{g_{ff}^2 \partial_r g_{tt}}{g_{tt}\partial_r g_{ff} - g_{ff}\partial_r g_{tt}} \tag{7}$$

and corresponding energy

$$\mathcal{E} = -g_{tt}\left(1 + \frac{\mathcal{L}_c^2}{g_{\phi\phi}}\right) \tag{8}$$

Below we analyze the effects of deviation parameters of both non & quasi=Schwarzschild black holes on the values of specific angular momentum and energy of particles at circular orbits.

Now, we study the effect of spacetime deformation on the innermost stable circular orbits (ISCOs) of the test particles. $(dr/d\tau = 0 \,\&\, p^r = 0)$ $V_{eff} = \varepsilon$ and $d^2r/d\tau^2 = 0$ or $dV_{eff}/dr = 0$, together with $\frac{d^2V_{eff}}{dr^2} = 0$.

The radius of the ISCO satisfies the conditions $V_{eff} = \varepsilon \; \frac{dV_{eff}}{dr^2} = 0$ and $\frac{d^2V_{eff}}{dr^2} = 0$ and to obtain the ISCO region we solve the non-linear system of equations. Due to the complicated form of the equations, we analyze the ISCO profiles graphically. In Figure 1, we demonstrate the dependence of the ISCO radius on the deviation parameter in the top left panel. In addition, in the rest of the panels, we analyze possible values of the particle energy and angular momentum at the ISCOs corresponding to the range $\epsilon, \varepsilon \in (-1,1)$ Our analyses have shown that there is an upper limit for the deviation parameter of quasi-Schwarzschild spacetime near $\epsilon_{upp} \simeq 0.8$, in which the ISCO radius takes a minimum value, while ε causes a linear decrease in ISCO in the given range. It is observed from the figure that the range of the ISCO radius, energy, and angular moment of particles in the ISCO in the quasi-Schwarzschild case is much larger than in the non-Schwarzschild case.

The Keplerian accretion disc around astrophysical black holes in the Novikov–Thorn model is a model for the geometrically thin disc governed by circular geodesic properties in spacetime.

They are accreting matter orbiting black holes resulting in the extraction of high-energy gravitational and electromagnetic radiation. The energy efficiency of the accretion disc is the maximum value of the released energy from the accretion disc. Falling matter consisting of test particles becomes ionized as they obtain a relativistic high speed and viscosity in the denser matter. The energy efficiency of an isolated particle is proportional to the total luminosity of the accretion disk in the optical band of the electromagnetic waves.

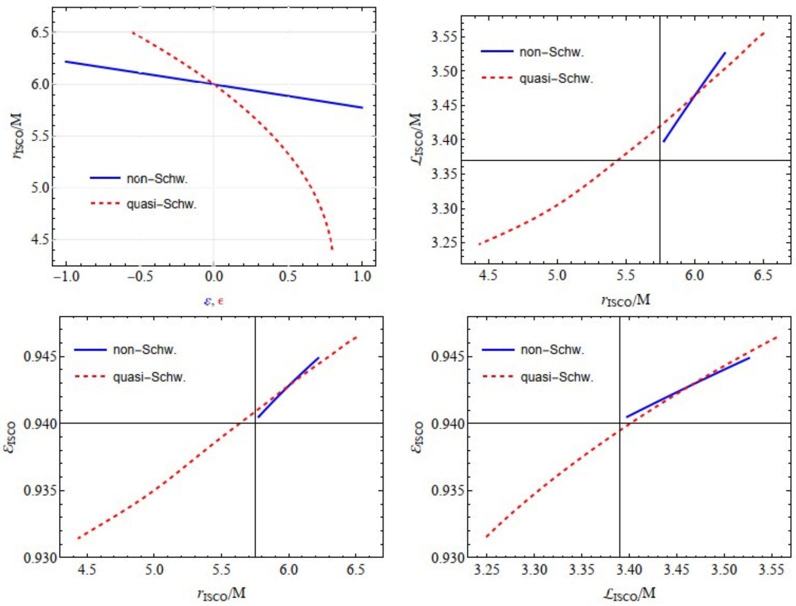

Figure 1. ISCO radius, energy, and angular momentum of test particles at ISCO.

The efficiency of energy release processes in the accretion disc is the difference between the rest energy of the test particles and their energy at ISCO

$\eta = 1 - \varepsilon_{ISCO}$, where $\varepsilon_{ISCO} = \varepsilon(r = r_{ISCO})$ describes the ratio of the binding energy of the particles with respect to the black hole particle system.

We investigate the energy efficiency around non-Schwarzschild and quasi Schwarzschild black holes. The obtained results have shown that the efficiency increases as the deformation parameters increase. The rate increases in the quasi-Schwarzschild black hole case concerning the non-Schwarzschild one

4. Conclusion

In this paper, we have focused on comparing the gravitational effects of deviation parameters of the spacetime of quasi & non-Schwarzschild black holes. To do this, we first study the dynamics of test particles in spacetime around both quasi & non-Schwarzschild black holes. We also compare the effects of spacetime deformation parameters of the black hole and black hole on the ISCO position, energy, and angular momentum of test particles corresponding to circular orbits and their values at ISCO together with the energy efficiency of the accretion disc around the black holes in the Novikov-Thorn model.

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